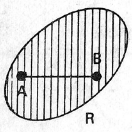
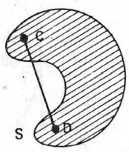
# Geometria Plana – I

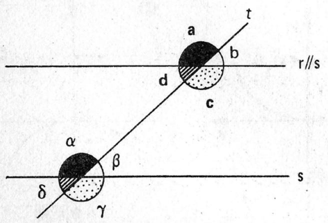
## 1. REGIÃO CONVEXA E NÃO CONVEXA

| convexa | não convexa |
|---|---|
|  |  |
| $\left.\begin{matrix}\forall A, B \in R \\ A \neq B\end{matrix}\right\} \Rightarrow \overline{AB} \subset R$ | $\exists C, D \in S \mid \overline{CD} \not\subset S$ |





## 2. PARALELISMO




a) **Ângulos Correspondentes:** $r // s \Leftrightarrow \alpha = a$

Analogamente: $\beta = b$; $\gamma = c$; $\delta = d$

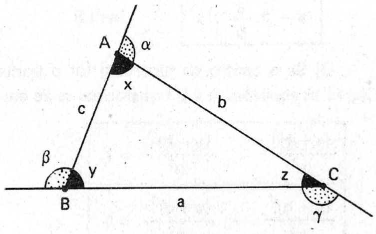
b) **Ângulos Alternos:** $r // s \Leftrightarrow \alpha = c$

Analogamente: $\beta = d$; $\gamma = a$; $\delta = b$

c) **Ângulos Colaterais:** $r // s \Leftrightarrow \alpha + d = 180^o$

Analogamente: $\beta + c = \gamma + b = \delta + a = 180^o$

## 3. TRIÂNGULOS




a) **Relações Angulares**

$x + y + z = 180^o$

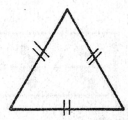
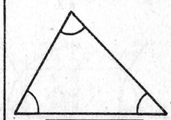
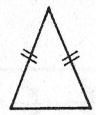
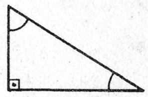
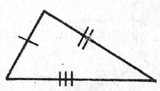
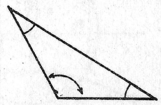
| | | |
|---|---|---|
| $\alpha = y + z$ | $\beta = x + z$ | $\gamma = x + y$ |

$\alpha + \beta + \gamma = 360^o$

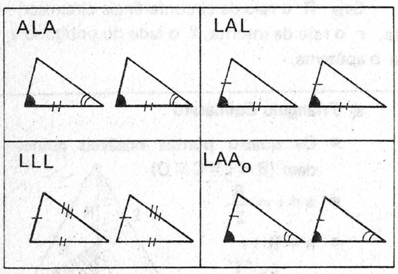
b) **Condições de Existência**

| | | |
|---|---|---|
| $a < b + c$ | $b < a + c$ | $c < a + b$ |

c) **Classificação**

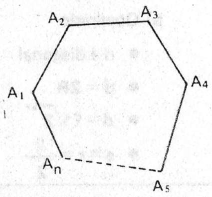
| Quanto aos lados | Quanto aos ângulos |
|---|---|
|  equilátero |  acutângulo |
|  isósceles |  retângulo |
|  escaleno |  obtusângulo |

d) **Critérios de Congruência**




4. **POLÍGONOS CONVEXOS DE n LADOS**

- $d = \frac{n(n-3)}{2}$
- $S_i = (n-2) \cdot 180^o$
- $S_e = 360^o$

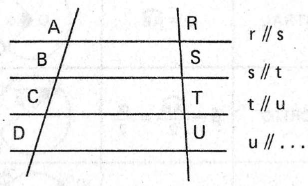



Se o polígono for regular então

a) Cada ângulo interno vale: $\frac{(n-2) \cdot 180^o}{n}$

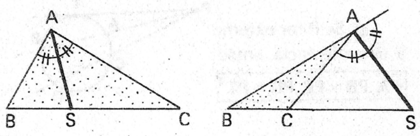
b) Cada ângulo externo vale: $\frac{360^o}{n}$

5. **TEOREMA DE TALES**



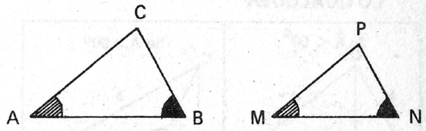

$r // s // t // u // \ldots \Rightarrow \frac{AB}{RS} = \frac{BC}{ST} = \frac{CD}{TU} = \ldots$

6. **TEOREMA DA BISSETRIZ**




$\vec{AS}$ **é bissetriz** $\Leftrightarrow \frac{AB}{BS} = \frac{AC}{CS}$

7. **SEMELHANÇA DE TRIÂNGULOS**




a) $\Delta ABC \sim \Delta MNP \Leftrightarrow \begin{cases} \hat{A} \cong \hat{M}, \hat{B} \cong \hat{N}, \hat{C} \cong \hat{P} \\ \frac{AB}{MN} = \frac{BC}{NP} = \frac{AC}{MP} = k \end{cases}$

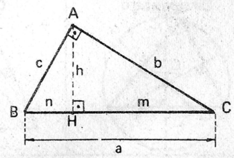
b) $\Delta ABC \sim \Delta MNP \Rightarrow \frac{\text{Área } (\Delta ABC)}{\text{Área } (\Delta MNP)} = k^2$

c) **Critérios:**

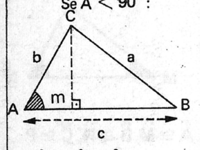
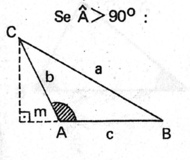
| | | |
|---|---|---|
| AA∼ | LAL ∼ | LLL∼ |

8. **RELAÇÕES MÉTRICAS NOS TRIÂNGULOS RETÂNGULOS**

- $a^2 = b^2 + c^2$ **(Teorema de Pitágoras)**
- $b^2 = a \cdot m$
- $c^2 = a \cdot n$
- $h^2 = m \cdot n$
- $b \cdot c = a \cdot h$



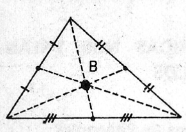
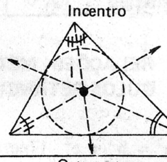
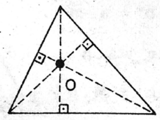
## 9. RELAÇÕES MÉTRICAS NUM TRIÂNGULO QUALQUER

| Se $\hat{A} < 90^o$: | Se $\hat{A} > 90^o$: |
|---|---|
|  |  |
| $a^2 = b^2 + c^2 - 2mc$ | $a^2 = b^2 + c^2 + 2mc$ |

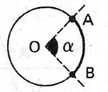
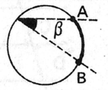
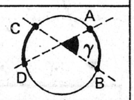
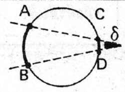
Calculando **m** em função de **b** e **cos A** obtemos, em ambos os casos, o teorema dos co-senos:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

## 10. PONTOS NOTÁVEIS DO TRIÂNGULO (BICO)

| Baricentro | Incentro |
|---|---|
|  |  |
| **Circuncentro** | **Ortocentro** |
|  |  |

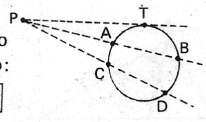
## 11. ÂNGULOS NA CIRCUNFERÊNCIA

| | | |
|---|---|---|
| CENTRAL | $\alpha = \widehat{AB}$ |  |
| INSCRITO | $\beta = \frac{\widehat{AB}}{2} = \frac{\alpha}{2}$ |  |
| EXCÊNTRICO INTERIOR | $\gamma = \frac{\widehat{AB} + \widehat{CD}}{2}$ |  |
| EXCÊNTRICO EXTERIOR | $\delta = \frac{\widehat{AB} - \widehat{CD}}{2}$ |  |

## 12. POTÊNCIA DE PONTO

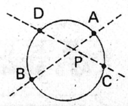
a) Se P for externo à circunferência, então:

$$PA . PB = PC . PD = PT^2$$



b) Se P for interno à circunferência, então:

$$PA . PB = PC . PD$$

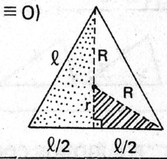


## 13. POLÍGONOS REGULARES

Seja **R** o raio da circunferência circunscrita, **r** o raio da inscrita, ℓ o lado do polígono e **a** o apótema.

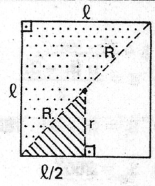
**a) Triângulo Eqüilátero**

- Os quatro pontos notáveis coincidem (B ≡ I ≡ C ≡ O)
- $a = r = \frac{R}{2}$
- $h = R + r$
- $h = \frac{\ell\sqrt{3}}{2}$



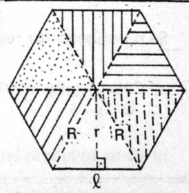
**b) Quadrado**

- d é diagonal
- $d = 2R$
- $d = \ell\sqrt{2}$
- $a = r = \frac{\ell}{2}$



**c) Hexágono Regular**

- Seis triângulos eqüiláteros
- $\ell = R$
- $a = r = \frac{R\sqrt{3}}{2}$

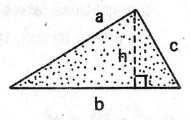
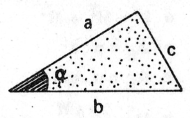
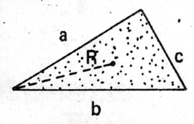
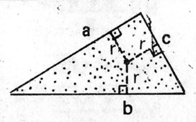
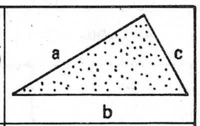
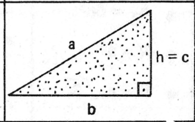
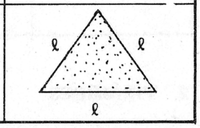


# Geometria Plana – IV

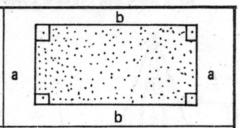
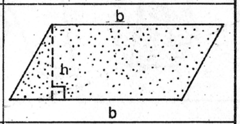
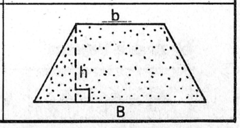
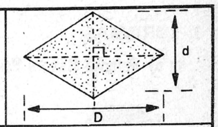
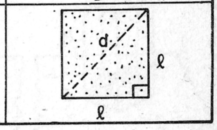
## 12. ÁREAS DAS FIGURAS PLANAS

### a) Triângulos

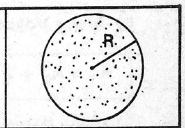
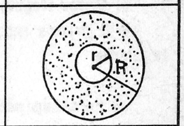
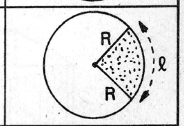
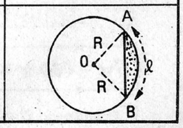
Sendo **R** o raio da circunferência circunscrita, **r** o da inscrita e $p = \frac{a+b+c}{2}$ o semiperímetro, a área de um triângulo pode ser calculada das seguintes formas:

| | |
|---|---|
| $S = \frac{b \cdot h}{2}$ |  |
| $S = \frac{ab \cdot \text{sen}\,\alpha}{2}$ |  |
| $S = \frac{a\,b\,c}{4R}$ |  |
| $S = p \cdot r$ |  |
| $S = \sqrt{p(p-a)(p-b)(p-c)}$ fórmula de Hierão |  |
| $S = \frac{b\,c}{2}$ |  |
| $S = \frac{\ell^2\sqrt{3}}{4}$ |  |

### b) Quadriláteros Notáveis

| | |
|---|---|
| **Retângulo:** $S = ab$ |  |
| **Paralelogramo:** $S = b\,h$ |  |
| **Trapézio:** $S = \frac{(B+b) \cdot h}{2}$ |  |
| **Losango:** $S = \frac{D\,d}{2}$ |  |
| **Quadrado:** $S = \ell^2 = \frac{d^2}{2}$ |  |

### c) Figuras Circulares

| | |
|---|---|
| **Área do círculo:** $S = \pi R^2$ **Comprimento da circunf.:** $\ell = 2\pi R$ |  |
| **Coroa Circular** $S = \pi(R^2 - r^2)$ |  |
| **Setor Circular:** $S = \frac{\ell\,R}{2}$ |  |
| **Segmento Circular:** $S = \frac{\ell\,R}{2} - S_{\triangle OAB}$ |  |